FROM TOPOLOGICAL FIELD THEORIES TO COVARIANT MATRIX STRINGS

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Abstract This paper is a shortened version of the previous work [2]: We propose a topological quantum field theory as a twisted candidate to formulate covariant matrix strings. The model relies on the octonionic or complexified instanton equations defined on an eight dimensional manifold with reduced holonomy. To allow untwisting of the model without producing an anomaly, we suggest (partially twisted) $\mathcal{W}$-gravity as an "extended" 2d-gravity sector.

The covariantization with respect to both worldsheet and spacetime symmetries of the orbifold theory of Dijkgraaf, Verlinde and Verlinde [1], if it can be achieved, should produce a covariant formulation of the second quantized free string theory. The statement that these covariant matrix strings exist is quite obvious in the commuting limit. Indeed, consider the superconformal theory constructed as the sum of N copies of the standard Neveu–Schwarz–Ramond (NSR) superstring. It has central charge $c = N \times 0$ and involves the following sets of fields: bosons X_i^μ, left and right moving fermions $\psi^\mu_{i;L,R}$ and $2d$ gravity ghosts $(b_i, c_i), (\beta_i, \gamma_i)$. The indices $\mu, \nu = 1 \cdots 10$ are acted on by the ten-dimensional Lorentz group, and the index $i = 1 \cdots N$ by the $\mathcal{S}_N$ symmetry group. This theory, whose action is simply the sum of N NSR superstrings' actions, can be "orbifolded" with respect to the $\mathcal{S}_N$ symmetry (add twisted sectors —the long strings— and project onto the invariant states). Orbifolding preserves both the manifest ten-dimensional and (less manifest)

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two-dimensional covariance, and the theory it leads to has all the nice features of a covariant string theory. It exhibits a $\mathcal{W}$-like chiral algebra, generated by the stress energy tensor $T = \sum_i T_i$ and its higher spin analogues $\mathcal{W}_{l=1\cdots N} = \sum_i T_i^l$. Furthermore, one can show that on the cylinder, the theory is equivalent to the Green–Schwarz theory of [1].

The tricky question is of course that of going away from the $g_{_\mathrm{s}} = 0$ limit. In [2], we make a step and address this issue from the "twisted" point of view. Namely, we propose a topological quantum field theory (TQFT) as a twisted candidate, possibly invariant under $SO(8)\times SO(2)$. The TQFT is based on the generalized octonionic or complexified instanton equations defined in eight dimensions for a $U(N)$ Yang–Mills field [3] *and* superpartners, living on an eight dimensional manifold $\mathcal{M}_8$ with special holonomy.

The octonionic instantons arise from the generalized self-duality equations that occur when $\mathcal{M}_8$ has $Spin(7)$ holonomy:

$$\mathcal{F}_{i8} \pm \frac{1}{2} c_{ijk}\mathcal{F}_{jk} = 0, \qquad \text{for} \quad i, j, \cdots = 1, \ldots, 7 \tag{1}$$

Where $\mathcal{F}$ denotes the curvature of the gauge field $\mathcal{A}$, and the c_{ijk} are the structure constants for octonions.
In the case of $\mathcal{M}_8$ being a Calabi–Yau fourfold, it is the $SU(4)$ instantons one deals with, obtained from

$$\begin{array}{c} \mathcal{F}_{z^A\bar{z}^A} = 0, \\ \mathcal{F}_{z^A z^B} \pm \epsilon_{_{ABCD}}\mathcal{F}_{\bar{z}^C\bar{z}^D} = 0, \end{array} \qquad \text{for} \quad A, B, \cdots = 1, \ldots, 4 \tag{2}$$

Such equations can be used as gauge functions to define topological field theories [4] in which the topological symmetry can possibly be untwisted into supersymmetry, in the same spirit as [5]. The theories of [3] have been constructed in this manner and, after reduction to two dimensions, provide a twisted version of the light-cone matrix strings of [1]. Their topological multiplet can be untwisted into the degrees of freedom of the light-cone matrix string under the following correspondence, inspired of [6] (according to the splitting $\mathcal{M}_8 = \mathcal{M}_6 \times \Sigma$ where Σ is two–dimensional and $\mathcal{M}_6$ is taken to be small, the gauge field $\mathcal{A}_{z^{A=1\cdots 4}}$ splits as $\phi_a \sim \mathcal{A}_{z^{a=1\cdots 3}}$ and $A_z \sim \mathcal{A}_{z^4}$):

$$\begin{array}{rcl} \phi^a,\, \bar{\phi}^a,\, \varphi,\, \bar{\varphi} & \rightarrow & X^{m=1\cdots 8}, \\ \Psi^a,\, \chi^{\bar{a}},\, \eta+\chi,\, \Psi_z & \rightarrow & \psi^m_{_L}, \\ \Psi^{\bar{a}},\, \chi^a,\, \eta-\chi,\, \Psi_{\bar{z}} & \rightarrow & \psi^m_{_R}. \end{array} \tag{3}$$

Here, for each classical field $(A_{z,\bar{z}}, \phi, \bar{\phi})$, the Ψ's are the topological ghosts and the χ's are the antighosts. The Yang–Mills field also involves ghosts for ghosts (the φ's), responsible for the gauge symmetry, and the associated fermionic Lagrange multiplier η. All fields are $U(N)$ valued; the ϕ^a's are complex and we note $\phi^{\bar a} \sim \bar\phi^a \sim \phi^{a\dagger}$.

To go beyond the light-cone version, we make two distinct steps. The first one, concerning ten-dimensional spacetime covariance, is to enhance the field content to that of the whole eight-dimensional Yang–Mills supermultiplet, to end with the degrees of freedom of a ten-dimensional target space. Hence, instead of constructing the TQFT for the sole $U(N)$ gauge field $\mathcal{A}_\mu$, we include its eight dimensional superpartners $\lambda^\alpha, \phi^\pm$.The scalars $\phi^\pm$ will provide the longitudinal coordinates, whereas the fermions λ^α (and topological partners) open the gate to a heterotic sector.

This in turn *naturally* leads to the second step, that of two dimensional covariance. Indeed, one needs to gauge fix the topological symmetry of the fermions and scalars, and this can only be done if we dimensionally reduce down to two dimensions. However, simplistic reduction turns our gauge functions (1)(2) into a set of inconsistent equations. Two-dimensional covariance implies the coupling to a topological $2d$ gravity system. The metric is introduced in the Beltrami parametrization (it allows for manifest left-right factorization [7]): $ds^2 = e_z^+ e_{\bar z}^-(dz + \mu_{\bar z}^z d\bar z)(d\bar z + \mu_z^{\bar z} dz)$ and the symmetries are used to set the background values of $e_z^+, e_{\bar z}^-$ to 1. To make our gauge functions covariant, we introduce additional fields, of the Wess–Zumino type. Denoted as $L, \bar L$, their conformal weight is fixed by the requirement that e^L and $e^{\bar L}$ have weight $(1,0)$ and $(0,1)$ respectively. Making use of these, we can define our two dimensional covariant gauge functions, based on (2), as follows:

$$\begin{array}{rcl} \mathcal{E} & \sim & F_{z\bar z} + e^{L+\bar L}\sum_a [\phi^a, \phi^{\bar a}] \\ \mathcal{E}^a_{\bar z} & \sim & D_{\bar z}\phi^a + \frac{1}{2} e^{\bar L}\varepsilon^{abc}[\phi^{\bar b}, \phi^{\bar c}] \\ \mathcal{E}^{\bar a}_z & \sim & D_z\phi^{\bar a} + \frac{1}{2} e^{L}\varepsilon^{abc}[\phi^{b}, \phi^{c}] \end{array} \tag{4}$$

with derivatives now covariant with respect to both the gauge invariance and the $2d$ gravity (they involve $A_z, A_{\bar z}$, the Beltrami's and the Christoffel's). As for the superpartners, we choose to impose the right-moving condition on the fermions and holomorphicity on the scalars. We have imposed two-dimensional covariance; we deal with the degrees of freedom of a ten dimensional target space theory via $X^{M=1\cdots 10} \leftarrow \{X^m, \phi^\pm\}$ for bosons, and $\psi_L^M \leftarrow \{\psi_L^m, \Psi^+, \chi^-\}$, $\psi_R^M \leftarrow \{\psi_R^m, \Psi^-, \chi^+\}$ for fermions.

The key point is then to check whether we are allowed to map our fields to physical ones as above. Indeed, it is crucial that we are able to untwist our (topological and thus anomaly free) theory into a supersymmetric one without producing an anomaly: Can we change the spins of our fields to the physical ones without spoiling the vanishing of the central charge? In the usual $U(1)$ superstrings, the light-cone

theory is a gauge fixed version where the longitudinal components are canceled by the $2d$ gravity ghost systems arising from two dimensional reparametrizations. For non-commuting strings, it is not clear how one can make sense of "matrix-valued" $2d$-diffeomorphisms, nor what kind of "extended" ghost system would be capable of compensating for the longitudinal $U(N)$ degrees of freedom (namely $2(N^2-1)$ non-physical bosons, and correspondingly for the fermions).
We refer the reader to [2], where we propose evidence that the appropriate ghost systems might arise from the $\mathcal{W}_{N+1}$ of $\mathcal{W}_\infty$ partially twisted gravity. Indeed, the TQFT based on the Lie algebra $\mathcal{W}_\infty$ has an infinite series of ghosts that could be partially untwisted to produce the set of gravitational ghost systems to compensate for the matrix string's matter anomaly. Different twists and gauge fixings might allow to get either a covariant NSR theory *or* heterotic matrix strings (via more complicated compensations involving the fermions λ^α, in the spirit of [6]).

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